Pearson Edexcel

Mark Scheme (Results)

January 2019

Pearson Edexcel International GCSE
in Further Pure Mathematics (4PM0)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks (dependent on the preceding M mark)
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**

If no working is shown then correct answers normally score full marks.

If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated always check the working in the body of the script and award any marks appropriate from the mark scheme.

  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

  Any case of suspected misread loses two A (or B) marks on that part, but can gain the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

- **Follow through marks**

  Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

  Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=\ldots.$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=\ldots.$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=\ldots.$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

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4PM0 Further Pure Mathematics Paper 1

Question number	Scheme	Marks
1	$27=\frac{1.5}{2}r^2 \Rightarrow r=\sqrt{36}=6$ (cm)	M1A1
	$l=6\times1.5=9 \Rightarrow P=6+6+9=21$ cm	M1A1 [4]

Additional Notes	
Mark	**Guidance**
M1	Uses correct formula for the area of the sector of a circle; $A=\frac{\theta}{2}r^2$ or $r^2=\frac{2A}{\theta}$ **and** substitutes A = 27 and $\theta=1.5$ correctly to achieve a value for r (must be r and not just r^2)
A1	r = 6
M1	Uses correct formula for the length of an arc of a circle; $l=r\theta$,substitutes in their value for r and $\theta=1.5$ correctly **and** adds 2 × their r to achieve a value for the perimeter.
A1	P = 21 (cm)
For a value of 21 cm without any working, award M1A1M1A1	

ALT – Works in degrees	
Mark	**Guidance**
M1	Changes 1.5 radians into degrees and uses correct formula for the area of the sector of a circle; 1.5 radians $=\frac{270}{\pi}=85.9...^\circ \Rightarrow 27=\frac{85.9...^\circ}{360}\times\pi\times r^2 \quad\Rightarrow r='6'$ (and achieves a value for r (must be r and not just r^2) The conversion of 1.5 radians to degrees must be correct.
A1	r = 6 Accept a value of r that rounds to 6.
M1	Uses correct formula for the length of an arc of a circle; $l=\frac{85.9...^\circ}{360}\times2\times\pi\times'6'='9'$ **and** adds 2 × their r to achieve a value for the perimeter.
A1	P = 21 (cm)
For a value of 21 cm without any working, award M1A1M1A1	

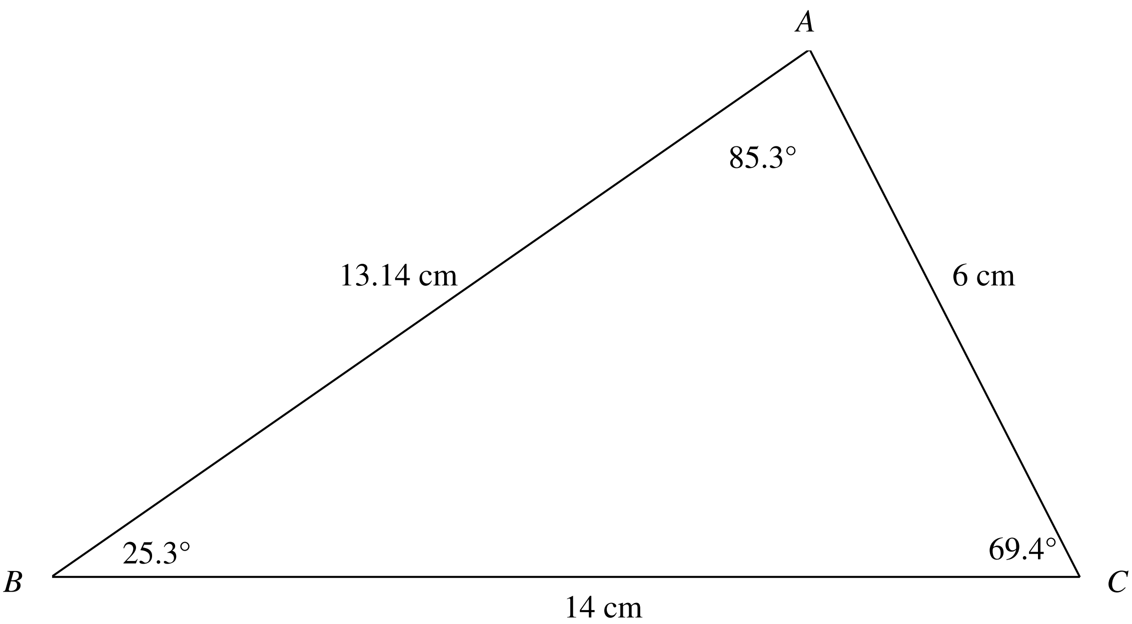
Question number	Scheme	Marks
2 (a)	$S_n = \sum_{r=1}^{n}(4r+1) \Rightarrow a=5 \quad d=4$	B1B1
	$S_n = \frac{n}{2}(2\times 5+(n-1)4) \Rightarrow S_n = n(3+2n)$	M1A1 (4)
	ALT	
	$S_n = \sum_{r=1}^{n}(4r+1) \Rightarrow a=5 \quad l=4n+1$	{B1B1}
	$S_n = \frac{n}{2}(5+4n+1) \Rightarrow S_n = n(3+2n)$	{M1A1} {4}
(b)	$S_{n+3} - S_n = 3(5+14\times 4) = 183$ $(n+3)(3+2(n+3))-(n)(3+2n) = 183 \Rightarrow 12n+27=183$ $\Rightarrow n=13$	B1 M1M1 A1 (4)
	ALT $S_{n+3} - S_n = t_{n+3} + t_{n+2} + t_{n+1}$ $t_{n+3} + t_{n+2} + t_{n+1} = 3t_{n+2}$ $3t_{n+2} = 3t_{15} \Rightarrow n+2=15$ $\Rightarrow n=13$	{B1 M1 M1 A1} {4}
		[8]

Additional Notes			
Part	**Mark**	**Guidance**	
(a)	B1	Either $a=5$ **OR** $d=4$	Can be embedded in their summation formula.
	B1	Both $a=5$ **AND** $d=4$	
	M1	Uses the correct summation formula with their values of a and d	
	A1	Simplifies the summation formula to achieve $S_n = n(3+2n)$ **This is a show question**- please check there are no errors in their working.	
ALT 1			
(a)	B1	Either $a = 5$ **OR** $l = 4n + 1$	Can be embedded in their summation formula.
	B1	Both $a = 5$ **AND** $l = 4n + 1$	
	M1	Uses the correct summation formula (first plus last) with their values of a and l	
	A1	Simplifies the summation formula to achieve $S_n = n(3+2n)$ **This is a show question**- please check there are no errors in their working.	
ALT 2			
(a)	B1	For writing $\sum_{r=1}^{n}(4r+1) = 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$	
	B1	For $\sum_{r=1}^{n}(4r+1) = \frac{4n(n+1)}{2} + n$ Expands $4\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} 1$	
	M1	For $\sum_{r=1}^{n}(4r+1) = 2n^2 + 2n + n = 2n^2 + 3n$	
	A1	$\sum_{r=1}^{n}(4r+1) = n(2n+3)$ **This is a show question**- please check there are no errors in their working	
(b)	B1	Finds the value of $t_{15} = 61$ or $3t_{15} = 183$	
	M1	Uses the given summation formula from (a) to form an equation in n. They can start from the summation formula $S_n = \frac{n}{2}(2a + [n-1]d)$ but it must be correct with either their, or the correct values of a and d. ft their t_{15} or $3t_{15}$ for this mark.	
	M1	Forms a **linear** equation in n – the correct equation is $12n + 27 = 183$ o.e ft their t_{15} or $3t_{15}$ for this mark.	
	A1	$n = 13$	
ALT			
(b)	B1	Writes down $S_{n+3} - S_n = t_{n+3} + t_{n+2} + t_{n+1}$	
	M1	$t_{n+3} + t_{n+2} + t_{n+1} = 3t_{15}$	
	M1	$t_{n+3} + t_{n+2} + t_{n+1} = 3t_{n+2} \Rightarrow 3t_{n+2} = 3t_{15} \Rightarrow n + 2 = 15$ They must reach a **linear** equation in n for this mark	
	A1	$n = 13$	

Question number	Scheme	Marks
3 (a)	$2x^3+11x^2-x-3=(2x+1)(x^2+5x-3)$ $(2x+1)(x^2+5x-3)=2x^3+10x^2-6x+x^2+5x-3=2x^3+11x^2-x-3$ *	M1A1 cso (2)
(b)	$f(x)=0\quad x=\dfrac{-5\pm\sqrt{(-5)^2-4\times1\times-3}}{2\times1}\Rightarrow x=\dfrac{-5\pm\sqrt{37}}{2}$	M1A1
	So roots of $f(x)=0$ are $x=-\dfrac{1}{2},\ 0.541,\ -5.541$	A1 (3) [5]

Additional Notes		
Part	**Mark**	**Guidance**
(a)	M1	Attempts to multiply out the two brackets. Minimally acceptable response is as follows; • They must show all 6 terms in their expansion. • They must achieve $2x^3$ and another 4 correct terms. Please check these carefully, this is a **show question.** Accept a list of terms (with correct signs)
	A1	For the correct answer as printed with no errors or omissions.
(b)	M1	This mark is for solving equation resulting from the quadratic factor. Uses the correct quadratic formula [or completes the square] correctly to find **two** roots. Allow one sign error only provided the quadratic formula is seen first. If they do not quote the formula first, the substitution must be correct.
	A1	For $x=\dfrac{-5\pm\sqrt{5^2-4\times1\times(-3)}}{2}$
	A1	For all three roots seen **rounded correctly to 3 decimal places**. $x=-\dfrac{1}{2},\ 0.541,\ -5.541$ They do not have to be on one line, but we do need to see in their answer all three roots in their working.

Question number	Scheme	Marks
4 (a)	$a\left(\sin x^{\circ}\cos 30^{\circ} - \sin 30^{\circ}\cos x^{\circ}\right) = b\left(\sin x^{\circ}\cos 30^{\circ} + \sin 30^{\circ}\cos x^{\circ}\right)$	M1
	$\Rightarrow a\left(\frac{\sqrt{3}}{2}\sin x^{\circ} - \frac{1}{2}\cos x^{\circ}\right) = b\left(\frac{\sqrt{3}}{2}\sin x^{\circ} + \frac{1}{2}\cos x^{\circ}\right)$	M1
	$\Rightarrow \sqrt{3}(a-b)\sin x^{\circ} = (a+b)\cos x^{\circ}$	dM1
	$\Rightarrow \tan x^{\circ} = \frac{(a+b)}{\sqrt{3}(a-b)}$	ddM1A1 (5)
(b)	$\frac{\sin(x-30)}{6} = \frac{\sin(x+30)}{14}$	M1
	$\tan x^{\circ} = \frac{14+6}{\sqrt{3}(14-6)} \Rightarrow x = 55.2849^{\circ} \approx 55.3^{\circ}$	M1A1
	Angles are; $\angle BAC = 85.284...^{\circ}, \angle ABC = 25.284...^{\circ}$ and $\angle ACB = 180 - 110.56...^{\circ} = 69.4^{\circ}$	B1 (4)
(c)	Area = $\frac{1}{2} \times 6 \times 14 \times \sin 69.43... = 39.3$ (cm^2)	M1A1 (2)
		[11]



Additional Notes		
Part	**Mark**	**Guidance**
(a)	M1	Uses the given trig expansion to expand $\sin\left(x-30^\circ\right)$ and $\sin\left(x+30^\circ\right)$ These expansions must be correct for this mark. Accept $a\left[\sin x\cos\left(-30^\circ\right)+\cos x\sin\left(-30^\circ\right)\right]$ for this mark **Condone poor/missing brackets and *a* or *b* even missing for this mark**
	M1	Uses the exact values of $\cos 30^\circ$ $\left(\frac{\sqrt{3}}{2}\right)$ and $\sin 30^\circ$ $\left(\frac{1}{2}\right)$ to leave an equation in sin *x*, and cos *x* as a minimum **Condone poor/missing brackets and *a* or *b* even missing for this mark**
	dM1	Simplifies their equation to give a minimally acceptable $k\left(a-b\right)\sin x^\circ=\left(a+b\right)\cos x^\circ$ where *k* is a constant. This mark is dependent on the first M mark
	ddM1	Uses the given identity for tan *A* to form a minimally acceptable $\tan x^\circ=\frac{\left(a+b\right)}{k\left(a-b\right)}$ using their *k* This mark is dependent on the first M mark and the previous M mark.
	A1	For the final correct given identity. **Note: This is a show question.** There must be no errors for the award of this final A mark
(b)	M1	Uses a correct sine rule (either way around) to form the equation $\frac{\sin\left(x-30\right)}{6}=\frac{\sin\left(x+30\right)}{14}$ This must be correct for this mark.
	M1	Uses the **given** identity for tan x° to form an equation. Also accept $\tan x^\circ=\frac{6+14}{\sqrt{3}\left(6-14\right)}\Rightarrow x=........^\circ$
	A1	For $x=55.3^\circ$ or better Allow recovery from $x=-55.3^\circ$ if they clearly state $x=55.3^\circ$ as final answer
	B1	$\angle ACB=69.4^\circ$ cao
(c)	M1	Uses the correct formula for the area of a triangle with the correct given values of 6 cm and 14 cm with their $\angle ACB$ or as given in **ALT** if they find length *AB*. **Note:** $\angle ACB=180^\circ-\left('55.3'-30\right)^\circ-\left('55.3'+30\right)^\circ$ **ALT** $A=\frac{1}{2}\times 6\times 13.14\times\sin 85.3...=\left(39.3\right)$ or $A=\frac{1}{2}\times 14\times 13.14\times\sin 25.3...=\left(39.3\right)$
	A1	$A=39.3\text{ (cm}^2\text{)}$
ALT – finds the perpendicular height and then uses half base × height.		

(c)	M1	Perpendicular height $= 6 \times \sin(69.4)^{\circ} = 5.61.....$ Area $= \dfrac{5.61.....\times 14}{2} = (39.3)$
	A1	$A = 39.3$ (cm^2)

Question number	Scheme	Marks
5 (a)	$2x^2 + 7x - 4 = 2\left(x + \frac{7}{4}\right)^2 - \frac{81}{8}$	M1M1A1
	Hence $A = 2, B = \frac{7}{4}$ and $C = -\frac{81}{8}$	(3)
	ALT	
	$A(x+B)^2 + C = Ax^2 + 2ABx + (AB^2 + C)$	{M1M1A1}
	$\Rightarrow A = 2, B = \frac{7}{4}$ and $C = -\frac{81}{8}$	{(3)}
(b)	(i) Min value of $\mathrm{f}(x) = -\frac{81}{8}$	B1ft
	(ii) $-\frac{7}{4}$	B1ft (2)
(c)	$2x^2 + 7x - 4 = px - 6 \Rightarrow 2x^2 + x(7-p) + 2 = 0$	
	$b^2 - 4ac > 0 \Rightarrow (7-p)^2 - 4 \times 2 \times 2 = p^2 - 14p + 33$	B1
		M1
	$p^2 - 14p + 33 = 0 \Rightarrow (p-11)(p-3) = 0 \Rightarrow$ c.v's $p = 11, 3$	dM1A1
	Outside region $p < 3$ or $p > 11$ Accept eg $\{p < 3 \cup p > 11\}$ Accept correct inequality shown on a number line	A1 (5) [10]

<table>
<tr><td colspan="4">Additional Notes</td></tr>
<tr><td>Part</td><td>Mark</td><td colspan="2">Guidance</td></tr>
<tr><td rowspan="3">(a)</td><td>M1</td><td colspan="2">Takes out a common factor of 2 out leaving
$2\left(x^2+\frac{7}{2}x\right)-4$ or $2\left(x^2+\frac{7}{2}x-2\right)$</td></tr>
<tr><td>M1</td><td colspan="2">Attempts to complete the square- minimally acceptable attempt is as follows:
$2x^2+7x-4=0=2\left(x\pm\frac{7}{4}\right)^2\pm q-4=0, \quad q\neq 0 \ \ p\neq 1$
This is an A mark in Epen2</td></tr>
<tr><td>A1</td><td colspan="2">$A=2, B=1.75$ and $C=-10.125$ or $A=2$ and $B=\frac{7}{4}$ and $C=-\frac{81}{8}$</td></tr>
<tr><td colspan="4">ALT compares coefficients</td></tr>
<tr><td rowspan="3">(a)</td><td>M1</td><td colspan="2">Multiplies out $A(x+B)^2+C$ to give $Ax^2+2ABx+\left(AB^2+C\right)$ (must be correct) AND compares with $2x^2+7x-4$ to attempt to find the values of A, B and C.
Minimum required is A = 2 for this mark.</td></tr>
<tr><td>M1</td><td colspan="2">Finds either $A=2$ and $B=\frac{7}{4}$ or $C=-\frac{81}{8}$
This is an A mark in Epen2</td></tr>
<tr><td>A1</td><td colspan="2">$A=2, B=\frac{7}{4}$ and $C=-\frac{81}{8}$</td></tr>
<tr><td rowspan="2">(b)</td><td>B1ft</td><td>$-\frac{81}{8}$ ft their C</td><td rowspan="2">Some candidates may differentiate having not achieved a correct part (a). Accept correct answers that do not follow through from their (a)</td></tr>
<tr><td>B1ft</td><td>$-\frac{7}{4}$ ft their $-B$</td></tr>
<tr><td rowspan="6">(c)</td><td>B1</td><td colspan="2">Sets $2x^2+7x-4=px-6$</td></tr>
<tr><td colspan="3">For the next 3 marks allow <, > or =</td></tr>
<tr><td>M1</td><td colspan="2">Forms 3TQ which must involve p in the term in x AND uses b^2-4ac with their 3TQ . b^2-4ac can be </>/= 0 for this mark.</td></tr>
<tr><td>dM1</td><td colspan="2">Solves their resulting 3TQ to find two critical values.
See general guidance for an attempt to solve a 3TQ by any method.

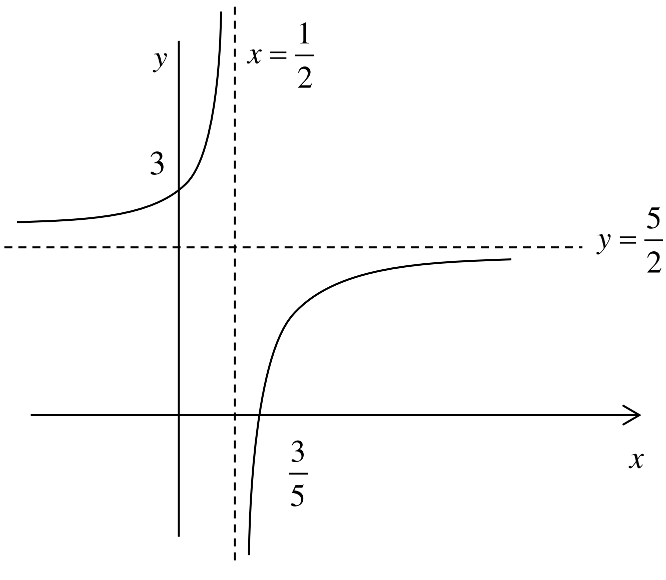
Accept correct use of letter in place of p for this mark</td></tr>
<tr><td>A1</td><td colspan="2">Correct critical values of 11 and 3 seen even if given in terms of x or another letter.</td></tr>
<tr><td>A1</td><td colspan="2">Correct inequality which must be in terms of p.</td></tr>
</table>

Question number	Scheme	Marks
6 (a)	$y = x^2\sqrt{(2x-3)} \Rightarrow \frac{dy}{dx} = x^2 \times (2x-3)^{-\frac{1}{2}} + (2x-3)^{\frac{1}{2}} \times 2x$	M1A1
	$\frac{dy}{dx} = \frac{5x^2-6x}{\sqrt{(2x-3)}} = \frac{x(5x-6)}{\sqrt{(2x-3)}}$ * cso	dM1A1 cso (4)
(b)	$x = 2 \Rightarrow \frac{dy}{dx} = \frac{2(10-6)}{1} = 8$	B1 (1)
(c)	Gradient of normal $= -\frac{1}{8}$	B1ft
	$y = 2^2\sqrt{(2\times 2-3)} = '4'$	B1
	$(y-4) = '-\frac{1}{8}'(x-2)$	M1A1ft
	$x + 8y - 34 = 0$	A1 (5) [10]

Additional Notes		
Part	**Mark**	**Guidance**
(a)	M1	An attempt to differentiate each term and to use product rule. Minimally acceptable attempt for the award of this mark is given below: $\frac{dy}{dx} = lx\sqrt{2x-3} + x^2k\left(2x-3\right)^{-\frac{1}{2}}$
	A1	Correct unsimplified $\frac{dy}{dx} = x^2 \times \left(2x-3\right)^{-\frac{1}{2}} + \left(2x-3\right)^{\frac{1}{2}} \times 2x$
	dM1	For an attempt to use a common denominator to simplify their $\frac{dy}{dx}$ Minimally acceptable attempt; $\frac{lx\sqrt{2x-3} \times \sqrt{2x-3} + x^2k}{m\left(2x-3\right)^{\frac{1}{2}}}$ where k, l and m are constants which must be consistent from their $\frac{dy}{dx}$. Do not accept incorrect work here. **This is an A mark in Epen2**
	A1	For the correct expression as shown in the question. cso **Note: This is a show question** – every step must be correct for the award of this mark.
(b)	B1	For $\frac{dy}{dx} = 8$
(c)	B1ft	For gradient of normal $= -\frac{1}{8}$ (Follow through their answer to part (b)
	B1	For $y = 4$
	M1	Uses either the formula correctly with their values of y and their **gradient of the normal** (their gradient of the normal cannot be their $\frac{dy}{dx}$) or uses $y = mx + c$ with their values for y and m. If they use $y = mx + c$ award this mark when they find a value for c.
	A1	For a correct equation in any form with the correct values
	A1	For the correct equation in the specified form. Accept any form with integer coefficients with all terms on one side; • $x + 8y - 34 = 0$, $8y + x - 34 = 0$ • $34 - x - 8y = 0$, $34 - 8y - x = 0$

Question number	Scheme	Marks
7(a)	Dimensions of box are $(16-2x)$ cm $(10-2x)$ cm and x cm	B1
	$V = x(16-2x)(10-2x) = 4x^3 - 52x^2 + 160x$ *	M1A1cso (3)
(b)	$\frac{dV}{dx} = 12x^2 - 104x + 160 = 0$	M1
	$12x^2 - 104x + 160 = (3x-20)(x-2) = 0$	
	$x = 2$ or $\frac{20}{3}$ (x cannot be $\frac{20}{3}$ as $10 - 2\times\frac{20}{3} = -\frac{10}{3}$ so $x = 2$)	dM1A1
	$\frac{d^2V}{dx^2} = 24x - 104 \Rightarrow \frac{d^2V}{dx^2} = 24\times 2 - 104 = -56$ { $-56 < 0$ hence max}	M1A1 (5)
(c)	$V = 4\times 2^3 - 52\times 2^2 + 160\times 2 = 144$	M1A1 (2) [10]

Additional Notes		
Part	**Mark**	**Guidance**
(a)	B1	Use the dimensions of the box as $(16-2x)$ cm, $(10-2x)$ cm and x cm
	M1	Finds an expression for the volume of the box using their dimensions. $V = x(16-2x)(10-2x)$ and attempts to multiply their expression out. [Accept $V = 4x^3 - 52x^2 + 160x$ without intermediate working]
	A1	For $(V =)4x^3 - 52x^2 + 160x$ only. **Note: This is a show question -** there can be no errors for the award of this mark.
(b)	M1	Differentiates the **given** expression for V **and** sets = 0. Their differentiated expression must be a 3TQ. See general guidance for the definition of an attempt.
	dM1	Solves their differentiated equation by any acceptable method and achieves two values for x. [If they only give $x = 2$ because they discard the other value then that is fine.]
	A1	$x = 2$ $\quad x = \frac{20}{3}$ needs to be discarded at some stage in their working for the award of this mark.
	M1	Finds the second derivative (again see General Guidance for the definition of an attempt) and substitutes either of their values of x to find a value for $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$
	A1	$\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = -56$ negative hence maximum. Cao.
	ALT for 2nd Derivative Some students may test the gradient either side of their $x = 2$ and come to a conclusion based on this.	
	M1	Tests **both sides** of their x For example $x = 1.5$ and $x = 2.5$ and find the gradient using their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	A1	Draws a correct conclusion. Eg., Gradient at $x = 1.5$ is positive, at $x = 2.5$ is negative hence going from positive to negative so maximum.
(c)	M1	Substitutes in their value of x (**which must be positive**) into the **given** expression for V and finds a volume.
	A1	$V = 144$ (coming from a correct value of x) If they also give an answer of −59.25 coming from substituting $\frac{20}{3}$ then award A0 **unless** they make it clear that the volume is 144.

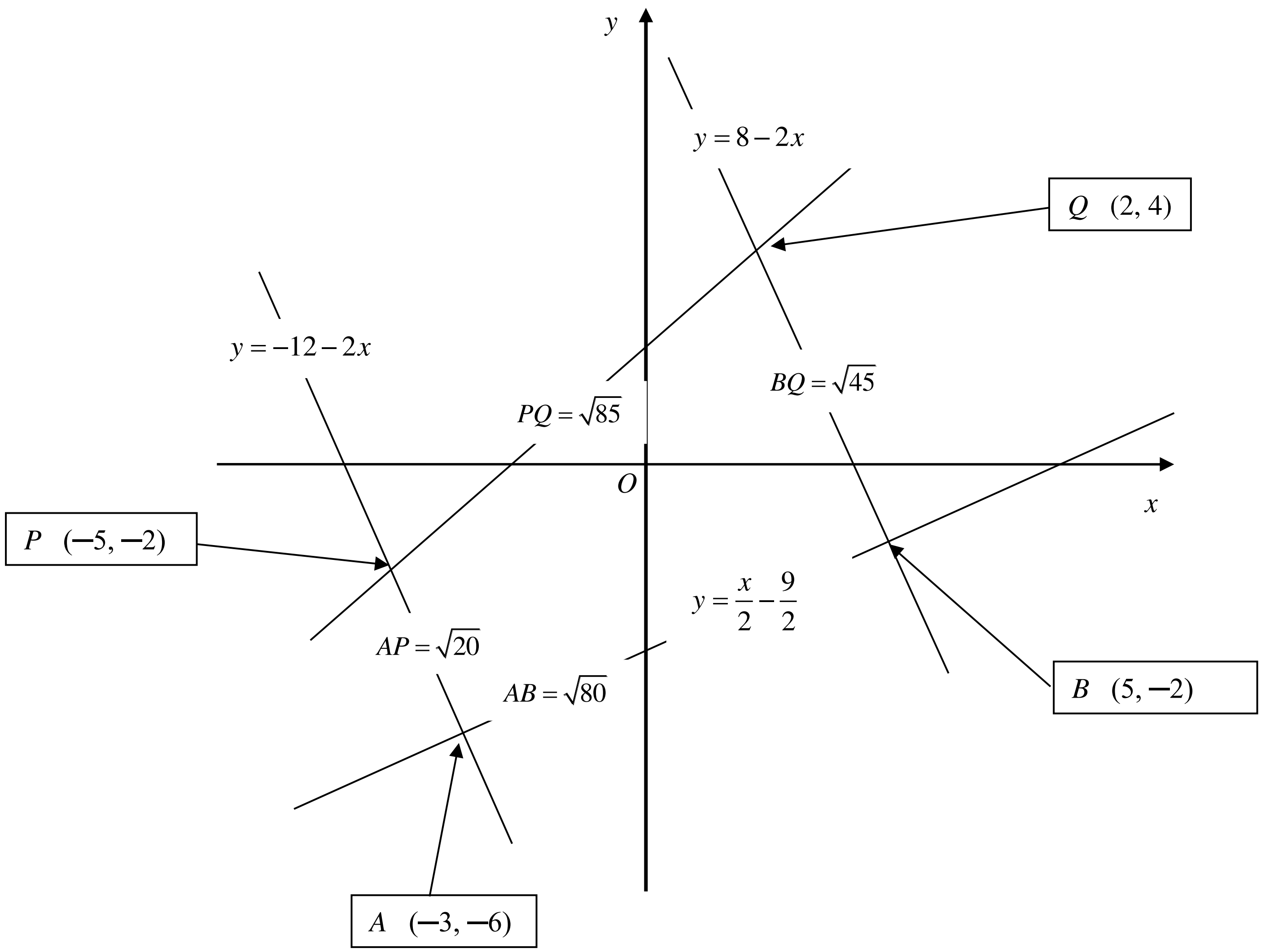
Question number	Scheme	Marks
8(a)	(i) $x=\frac{1}{2}$	B1
	(ii) $y=\frac{5}{2}$	B1 (2)
(b)	Coordinates of intersection with y-axis $y=\frac{-3}{-1}=3$ $(0,3)$	B1
	Coordinates of intersection with y-axis $5x-3=0 \Rightarrow x=\frac{3}{5} \Rightarrow \left(\frac{3}{5},0\right)$	B1
		(2)
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5(2x-1)-2(5x-3)}{(2x-1)^2}$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{(2x-1)^2}$ - numerator is positive, denominator is squared so always positive, hence $\frac{\mathrm{d}y}{\mathrm{d}x}$ is also always positive (except when $x=\frac{1}{2}$, so not defined).	dM1A1 (4)
(d)		B1 B1 B1 (3)
(e)	$y=2$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1^2}=1$	M1
	$(y-2)=1\times(x-1)$	M1
	$\Rightarrow y=x+1$	A1 (4)
		[15]

Additional Notes			
Part	**Mark**	**Guidance**	
(a) (i)	B1	$x = \frac{1}{2}$ or $x = 0.5$ only	If these parts are not labelled clearly (i) and (ii) then mark in order treating the first answer as (i) and the second as (ii)
(ii)	B1	$y = \frac{5}{2}$ or $y = 2.5$ only	
(b)	B1	$(0,3)$ OR $\left(\frac{3}{5},0\right)$	Accept **paired** values of $x = 0, y = 3$ $x = \frac{3}{5}, y = 0$
	B1	$(0,3)$ AND $\left(\frac{3}{5},0\right)$	
(c)	M1	Attempts to differentiate both terms and uses the quotient rule correctly. Minimally acceptable attempt; • $5x-3 \Rightarrow a, \quad 2x-1 \Rightarrow b \quad a \neq 0, b \neq 0$ • $\frac{dy}{dx} = \frac{a(2x-1)-b(5x-3)}{(2x-1)^2}$ OR $\frac{b(5x-3)-a(2x-1)}{(2x-1)^2}$ Or uses Product Rule with the same conditions.	
	A1	For the correct differentiated expression. $\frac{dy}{dx} = \frac{5(2x-1)-2(5x-3)}{(2x-1)^2}$	
	dM1	Simplifies their $\frac{dy}{dx}$ to give $\frac{dy}{dx} = \frac{k}{(2x-1)^2} \quad k \neq 0$	
	A1	A conclusion that must include; Denominator is squared hence will always be positive or $\frac{dy}{dx} > 0$	
(d)	B1	Shape: As shown with two arms, one arm in the 1st and 2nd quadrants, and the other arm in the 4th and 1st quadrants. The ends must look like they approach asymptotes – do not accept ends curling back on themselves.	
	B1	For **both** asymptotes drawn and marked cao	At least one branch of the curve must be present for the award of either of these marks (which must be asymptotic) and the curve must go **through** the axes for the intersection mark. i.e., not stop at the axis.
	B1	For **both** intersections marked cao	
(e)	B1	For $y = 2$ only	
	M1	Substitutes $x = 1$ into their differentiated expression and attempts to find a value for the gradient.	
	M1	Uses the formula for the equation of a line or $y = mx + c$ with their values of y and m. If they use $y = mx + c$ they must achieve a value for c for the award of this mark.	
	A1	For $y = x + 1$ or $y = 1 + x$	

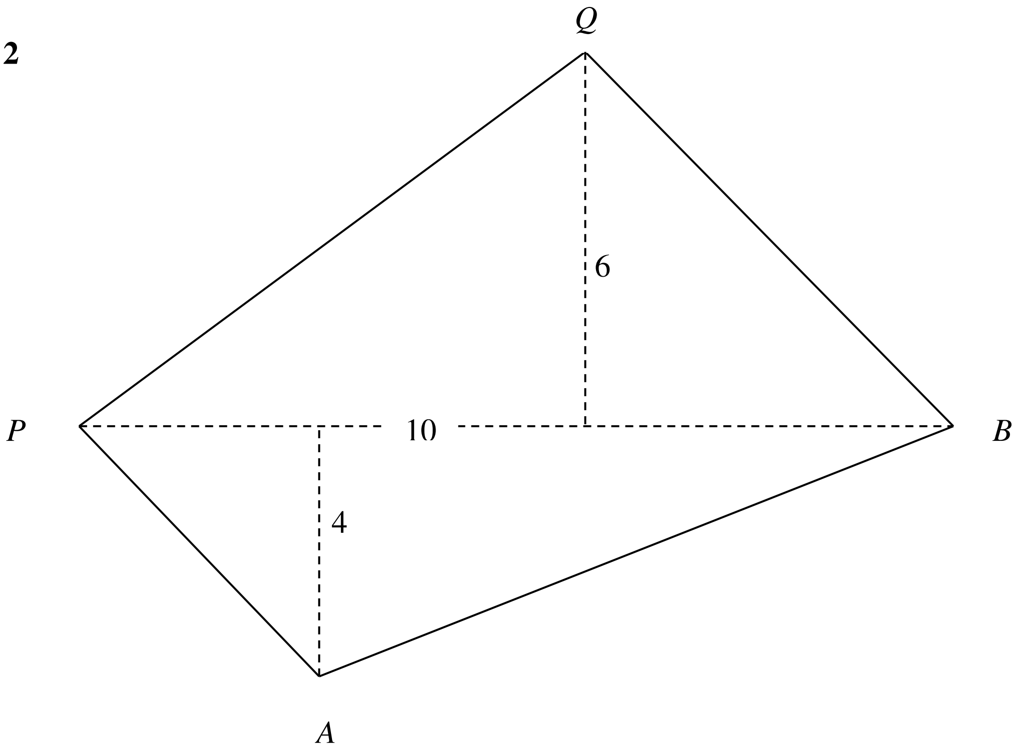
Question number	Scheme	Marks
9 (a)	$\dfrac{y--6}{-2--6}=\dfrac{x--3}{5--3} \quad \Rightarrow y=\dfrac{1}{2}x-\dfrac{9}{2}$ oe	M1A1A1 (3)
(b)	Gradient of perpendicular = −2	B1
	$y--6=-2(x--3)\Rightarrow -2=-2\times x-12\Rightarrow x=-5$ *	M1A1 cso (3)
(c)	Equation of perpendicular to *l* through *B*	B1
	$y--2=-2(x-5)\Rightarrow y=-2x+8$	
	$\sqrt{85}=\sqrt{(f--2)^2+(e--5)^2}$	M1
	$85=((-2e+8)+2)^2+(e+5)^2 \Rightarrow 0=5e^2-30e+40$	M1 M1A1
	$\Rightarrow (e-4)(e-2)=0\Rightarrow e=2,\quad (e=4)$	
	$f=-2\times 2+8\Rightarrow f=4$ Coordinates of Q are (2, 4)	A1 (6)
	ALT	
	$-2=\dfrac{f+2}{e-5}\Rightarrow f=8-2e$	
	$\sqrt{85}=\sqrt{(f--2)^2+(e--5)^2}$	[B1
	$85=((-2e+8)+2)^2+(e+5)^2 \Rightarrow 0=5e^2-30e+40$	M1
	$\Rightarrow (e-4)(e-2)=0\Rightarrow e=2,\quad (e=4)$	M1
	$f=8-2\times 2=4$ Coordinates of Q are $(2,\ 4)$	M1A1 A1 (6)]
(d)	Area of *ABQP*	
	$AP=\sqrt{(-3--5)^2+(-6--2)^2}=\sqrt{20}$	
	$AB=\sqrt{(-3-5)^2+(-6--2)^2}=\sqrt{80}$	
	$BQ=\sqrt{(2-5)^2+(4--2)^2}=\sqrt{45}$	
	Shape is a trapezium	M1 (either) A1 (all)
	Area $=\dfrac{1}{2}\left(\sqrt{80}\right)\left(\sqrt{20}+\sqrt{45}\right)=50$	M1A1 (4)
	ALT	
	$\dfrac{1}{2}\begin{pmatrix}-3 & 5 & 2 & -5 & -3\\ -6 & -2 & 4 & -2 & -6\end{pmatrix}=$	{M1A1
	$\dfrac{1}{2}\left((6+20-4+30)-(-30-4-20+6)\right)=50$	M1A1} (4) [16]

<table>
<tr><td colspan="3">Additional Notes</td></tr>
<tr><td>Part</td><td>Mark</td><td>Guidance</td></tr>
<tr><td>(a)</td><td>M1</td><td>Either uses the correct formula substituting in the correct values of y and x to form an equation,
OR finds the gradient of l using $m=\frac{y_1-y_2}{x_1-x_2}$ and then uses either
$y-y_1=m\left(x-x_1\right)$ or $y=mx+c$
If they use $y=mx+c$ they must reach a value of c for the award of this mark.
Allow one sign error.</td></tr>
<tr><td></td><td>A1</td><td>For the correct equation of the line un-simplified.</td></tr>
<tr><td></td><td>A1</td><td>For $y=\frac{1}{2}x-\frac{9}{2}$ oe</td></tr>
<tr><td>(b)</td><td>B1</td><td>For the gradient of the perpendicular of -2</td></tr>
<tr><td></td><td>M1</td><td>For a complete method to find the value of k.
Finds the equation of the line through A and P and subs in -2 to find the value of k.
$\left\{y--6=-2(x--3)\Rightarrow -2=-2\times x-12\Rightarrow x=-5\right\}$</td></tr>
<tr><td></td><td>A1</td><td>$k=-5$ Accept $x=-5$
Note: This is a show question – Every step must be correct for the award of this mark.</td></tr>
<tr><td colspan="3">ALT Uses the gradient of AP</td></tr>
<tr><td>(b)</td><td>B1</td><td>For the gradient of the perpendicular of -2 seen explicitly or used correctly</td></tr>
<tr><td></td><td>M1</td><td>For a complete method to find to find the value of k using gradients.
$-2=\frac{-2--6}{k--3}\Rightarrow -2k-6=4\Rightarrow k=\left(-5\right)$</td></tr>
<tr><td></td><td>A1</td><td>$k=-5$
Note: This is a show question – Every step must be correct for the award of this mark.</td></tr>
<tr><td>(c)</td><td>B1</td><td>Either writes down the equation of the line BQ using the coordinates of B
$y--2=-2\left(x-5\right)\Rightarrow y=-2x+8\Rightarrow\left(f=-2e+8\right)$
Or writes down an expression for the gradient of BQ using the coordinates of B
$-2=\frac{f+2}{e-5}\Rightarrow f=8-2e$</td></tr>
<tr><td></td><td>M1</td><td>Uses Pythagoras theorem with the coordinates of P and Q to form
$\sqrt{85}=\sqrt{\left(f--2\right)^2+\left(e--5\right)^2}$ or $85=\left(f--2\right)^2+\left(e--5\right)^2$</td></tr>
<tr><td></td><td>M1</td><td>Substitutes an expression for f to form a 3TQ in e
$\left(0=5e^2-30e+40\right)$</td></tr>
<tr><td></td><td>M1</td><td>Solves their 3TQ by any method (see general guidance) to achieve two values</td></tr>
<tr><td></td><td>A1</td><td>$e=2$ (and 4) | Accept coordinates.</td></tr>
<tr><td></td><td>A1</td><td>$f=4$ [$f>0$]</td></tr>
<tr><td colspan="3">ALT</td></tr>
</table>

(c)	B1	**Either** writes down the equation of the line *BQ* using the coordinates of *B* $y--2=-2(x-5) \Rightarrow y=-2x+8 \Rightarrow (f=-2e+8)$ **Or** writes down an expression for the gradient of *BQ* using the coordinates of *B* $-2=\frac{f+2}{e-5} \Rightarrow f=8-2e$
	M1	Uses Pythagoras theorem with the coordinates of *P* and *Q* to form $\sqrt{85}=\sqrt{(f--2)^2+(e--5)^2}$
	M1	**S**ubstitutes an expression for *e* to form an equation in *f* $85=(f+2)^2+\left(9-\frac{f}{2}\right)^2 \Rightarrow 0=5f^2-20f$ oe
	M1	Solves their 2TQ equation
	A1	$f=4$
	A1	$e=2$ $(f>0)$
Any attempt using ratios – please send to review.		
(d)	M1	Uses Pythagoras theorem to find the lengths of *AB* **or** *AP* **or** *BQ* $AB=\sqrt{(-3-5)^2+(-6--2)^2}=(\sqrt{80})$, $AP=\sqrt{('5'--3)^2+(-2--6)^2}=(\sqrt{20})$ $BQ=\sqrt{(5-2)^2+(-2-4)^2}=(\sqrt{45})$
	A1	$AB=\sqrt{80}$, $AP=\sqrt{20}$ and $BQ=\sqrt{45}$ **(ALL THREE)**
	M1	Shape is a trapezium so uses correct formula, or breaks down into a triangle and rectangle to find an area $A=\frac{1}{2}(\sqrt{'80'})(\sqrt{'20'}+\sqrt{'45'})=(50)$ **or** Area of rectangle = $\sqrt{'20'}\times\sqrt{'80'}='40'$ Area of triangle = $\frac{1}{2}(\sqrt{'45'}-\sqrt{'20'})\times\sqrt{'80'}='10'$ Total area = (50)
	A1	$A=50$
ALT 1		
(d)	M1	Uses correct formula for the area of the quadrilateral using determinants using their values of (e, f) There must be 5 sets of coordinates in either clockwise or anticlockwise order starting and finishing at the same coordinate
	A1	For the correct formula with the correct coordinates.
	M1	For processing the calculation correctly.
	A1	$A=50$ If they leave the final answer as $A=-50$ withhold this mark
ALT 2		
(d)	M1	For the area of either *APB* or *PBQ* _ $APB=\frac{6\times10}{2}=30$ $PBQ=\frac{4\times10}{2}=20$
	A1	For both correct areas of triangles *APB* and *PBQ*
	M1	For adding together their areas of triangles *APB* and *PBQ*
	A1	50


y
$y = 8 - 2x$
Q (2, 4)
$y = -12 - 2x$
$BQ = \sqrt{45}$
$PQ = \sqrt{85}$
O
x
P (−5, −2)
$y = \frac{x}{2} - \frac{9}{2}$
$AP = \sqrt{20}$
$AB = \sqrt{80}$
B (5, −2)
A (−3, −6)

ALT 2


Q
6
P
10
B
4
A

Question number	Scheme	Marks
10 (a)	$(1-2x)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)(-2x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^2}{2!}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(-2x)^3}{3!}$ $\Rightarrow (1-2x)^{-\frac{1}{2}}=1+x+\frac{3}{2}x^2+\frac{5}{2}x^3+\ldots$	M1A1A1 (3)
(b)	$-\frac{1}{2} \leqslant x<\frac{1}{2}$ or $\lvert x\rvert<\frac{1}{2}$ (Allow $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$ or $\lvert x\rvert \leqslant \frac{1}{2}$)	B1 (1)
(c)	$\left(2-x^2\right)\left(1+x+\frac{3}{2}x^2+\frac{5}{2}x^3\right)=2+2x+2x^2+4x^3$	M1M1A1 (3)
(d)	$\int_0^{0.2}\frac{\left(2-x^2\right)}{\sqrt{(1-2x)}}\,\mathrm{d}x=\int_0^{0.2}2+2x+2x^2+4x^3\,\mathrm{d}x=\left[2x+x^2+\frac{2}{3}x^3+x^4\right]_0^{0.2}$ $=(0.4+0.04+0.00533+0.0016)=0.4469$	M1M1 M1A1 (4) [11]

Additional Notes

Part	Mark	Guidance
(a)	M1	An attempt at the binomial expansion which must have as a minimum; • The first term is 1 • The denominators in terms 2, 3 and 4 are correct. • The power of x is correct in each term $\left(-2x,\ [-2x]^2,\ [-2x]^3\right)$ • $-2x$ is used correctly at least once.
	A1	The first term and at least one term in x correct and simplified
	A1	All terms correct and simplified. $1+x+\frac{3}{2}x^2+\frac{5}{2}x^3$
(b)	B1	Correct inequality $-\frac{1}{2} \leqslant x < \frac{1}{2}$ or $\lvert x\rvert < \frac{1}{2}$ Allow $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$ or $\lvert x\rvert \leqslant \frac{1}{2}$ isw other attempts when a correct range is seen.
(c)	M1	Shows that they intend to multiply their expansion in (a) by $\left(2-x^2\right)$
	dM1	Multiplies out the two brackets to at least 4 terms up to and including the term in x^3 with a constant, term in x and a term in x^2
	A1	For the fully correct expansion as shown. $2+2x+2x^2+4x^3$
(d)		**In part (d) the question clearly states using algebraic integration** **No evidence of algebraic integration – no marks**
	M1	For an attempt to integrate their expansion in (c) provided it is as a minimum a constant term and at least two algebraic terms. Ignore the limits for this mark
	A1	**This is an M mark in Epen** For a fully correct integrated expression as shown (ignore limits)
	dM1	Attempts to evaluate their integrated expression using the correct limits Substitution of 0 need not be seen
	A1	Area = 0.4469

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